AllianceDEX v1.0 – White Paper

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3rd November 2021

Abstract

AllianceBlock proposes a new approach to automated market making (AMM) by synchronizing two constant function market makers (CFMM). The methodology combines the advantages of individual CFMM and reduces the downside risk for liquidity providers (LPs). It is particularly suitable for assets with high volatility that can induce high impermanent loss for LPs. This risk can counterbalance the benefit of liquidity mining, that is the earning of associated trading fees. This new approach provides a way to largely limit the impact of such risk. A detailed comparison of the new AMM to other well known decentralized exchanges such as Uniswap and Balancer is performed in this article.

1 Introduction

With the recent surge of Decentralized Finance (DeFi), a variety of blockchain-powered applications aimed at creating decentralized alternatives to traditional financial services have emerged. With very large accessibility, DeFi brings benefits such as greater transparency, decentralization, enhanced security and peer-to-peer global transactions among many other advantages as discussed e.g. in [10]. The underlying mechanics rely on smart-contract technologies and equivalents. More specifically, the emergence of the Ethereum blockchain [9], which allows the encoding of arbitrary smart-contract functionalities and execution on a blockchain has been a catalyst for DeFi applications. DeFi reached impressive amounts of capital, with 100 billion USD in September 2021 up from 20 billion USD within a year, see [1]. The industry proves to be in constant expansion and provides continuous innovation.

DeFi has allowed its users to lend or borrow with services on the blockchain, see for example [24, 3], among others. A crucial DeFi application is the decentralization of asset exchanges. Until recently, exchanging digital

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and traditional assets was only feasible on classic systems that share an accepted and common design known as a continuous-limit order-book, see e.g. [18] for details. Such an order-book consists of a list of all bids and offers from buyers and sellers in the system, i.e. prospective buyers place a limit buy order, which specifies a maximum price at which they are willing to buy an asset. Other types of orders such as market order or stop loss are usually available depending on the Centralized Exchange (CEX). While offering a range of advantages, CEXs have also experienced problems ranging from high-profile thefts, some of which are reviewed in [8], to offenses such as price manipulation [16].

Decentralized exchanges (DEXs), improve several aspects of CEXs, for example, security vulnerabilities, centralized control of assets, custodian challenges and more as reported in [10]. There have been interesting designs to decentralize, at least partially, the continuous limit order-book. For example, a counter-party can select an order in the order book and present it to the smart contract with a signed counter-order. The smart contract executes the order and counter-order, clearing the order from the order book. In this model traders themselves perform order matching, an approach that has been used for example by Etherdelta. Similar off-chain trade matching with on-chain settlement enforced by smart contracts has been proposed by dYdX [23] and IDex [22] among others, where the exchange itself performs the matching of orders. dYdX [23] has, for example, seen large volume increases in recent market setups. Fully decentralized exchanges have been recently built upon protocols acting as Automated Market Makers (AMM). AMMs are algorithmic agents that provide liquidity in electronic markets, a topic that has been well studied in algorithmic game theory, see [27], and for which an early work is the logarithmic market scoring rule introduced in [19].

The first fully decentralized exchanges for digital assets have been built around Constant Function Market Maker models (CFMM), see e.g. [20, 4, 26, 12]. The mechanism links two or more reserves of the different participating assets dynamically, relying on a driving constant function with specific properties. The liquidity available on the reserves and the CFMM function then jointly determine the market price of any two assets.

Liquidity providers (LPs) in DEXs can generate revenues by providing their funds as liquidity to the DEX of their choice, which will allow traders to exchange the assets of interest. In other words the liquidity provided will allow trading of the digital assets to be fully handled, in a decentralized manner, on the blockchain. Funds provided by liquidity providers are protected because the custody and exchange logic is processed and guaranteed by the smart contract directly.

Traders will generally pay a trading fee for each exchange, which is then shared among the liquidity providers and represents a direct remuneration for the funds provided. Over time, the trading fees get accumulated and LPs can see substantial return on their capital. However, as a counterpart for that reward, LPs also face a risk associated to the change of spot price value, commonly referred to as impermanent loss (IL). The term "impermanent" is employed in the field since, without withdrawal, the LP has a non-zero probability to recover this loss if the spot reverts back to its initial value. An intuitive view of the phenomenon is when the market moves heavily, a LP can recover through the DEX dynamics, more of the cheaper assets and less of the valuable assets than when he/she entered the AMM. In practice however, trading fees accumulated during that time can counterbalance the impermanent loss if the LP holds his/her position long enough in the DEX and if the market does not deviate too drastically. Because of the last point, this risk therefore remains a main concern for LPs and an area of innovation among various participants in the DEX industry. Uniswap and associated constant product two-assets CFMM [4], represent one of the most widely used DEX by volume as of today. However, it also embeds an impermanent loss profile which can strongly negatively impact the returns of the liquidity provider. In Uniswap V3 [5], the authors provide a way to optimize liquidity and therefore mitigate indirectly the risk of impermanent loss by increasing returns from trading fees in a designated range of spot chosen by the LP. However, the intrinsic properties of the underlying Uniswap CFMM being unchanged, the impermanent loss impact can worsen if the spot exits the fixed range where the liquidity was concentrated.

A detailed analysis of the Uniswap V2 market maker is provided in [8]. In [26], Balancer has generalized the Uniswap formula by introducing the geometric mean market maker CFMM. A precise analysis of geometric mean markets can be found for example in [14, 6, 7]. Geometric mean markets with asymmetric weights can improve the impermanent loss profile and therefore increase the returns of LPs on either the rise or the fall of the spot, but not both. Indeed, the IL on one side of the spot deviation stays uncovered as with Uniswap and slippage properties for traders are significantly worse on one trade direction as well. As in [5], there are also different approaches which do not directly amend the impermanent loss profile of the AMM itself. Bancor V2 [21] provides a system of insurance pools as well as a protection of impermanent loss, paid-out with their own protocol's token. It is an efficient solution in most market regimes, however, as discussed in detail in [25], this methodology presents a systemic risk under a stressed market scenario; more precisely there is an exposure to a downward spiral riskthat can highly affect both liquidity providers and the protocol's token holders. Dodo [11] relies fully on external price oracles, such as ChainLink [13], to create a market maker algorithm. One of the main advantages of Dodo, similarly to Bancor V2, is to allow single sided liquidity provisioning by construction. The impermanent loss is also improved, however, liquidity providers bear inventory risks that have similar disadvantages to impermanent loss, particularly in highly volatile market conditions. Additionally, market making with external oracles may raise issues for non-liquid assets.

Some analyses have discussed the returns and impermanent loss of liquidity providers with some possibilities to statically or dynamically hedge the IL risk. For example [14, 7] analyse LPs' returns under geometric mean markets and CFMM more generally and introduce some hedging possibilities. In this paper we propose to work directly on the intrinsic mechanism of the automated market maker with a new approach in order to reduce the impermanent loss. This new methodology has advantages in that it can be used in combination with other non-AMM specific improvements of the impermanent loss, such as the ones proposed in [21, 5] to only cite a few. Our approach, which we denominate as sync-AMM standing for Synchronized Automated Market Maker, proposes to combine the properties of two, or possibly more, CFMM and therefore obtain improved joint-properties. The synchronization process allows to align the spot prices of the CFMMs at play for each new trade. Numerical results show that even under highly volatile scenarios, the LPs returns are significantly improved compared with other CFMMs. In the market setup tested and over the paths analyzed, the liquidity providers simulated in our test case, were able to obtain positive returns from trading fees compared to a buy-and-hold strategy with spot deviations rising and falling by a factor of 150.

The remainder of this paper is organized as follows. In Section 2, we define the automated market market models of interest as well as discuss the definition of standard nomenclature of the field. Additionally, we provide a summary of the analysis of the impermanent loss and slippage in geometric mean markets from [15]. Secondly, in Section 3, we discuss the main contribution of the paper and a new approach to AMM which provides a considerably improved impermanent loss profile. Additionally, Section 4 provides details on the simulation framework used for the testing of the AllianceDEX sync-AMM as well as showcase its impermanent loss profile under different market scenarios. Finally, Section 5 concludes with a brief summary of the contributions.

1.1 Disclaimer

The results discussed by the authors in this article do not constitute, in any form, an investment advice in the associated AllianceBlock decentralized exchange or any other mentioned decentralized exchange. The authors and AllianceBlock are not responsible for any loss incurred as a result of the use of the AllianceBlock decentralized exchange or any information discussed and reported in this paper. This article is meant to provide informational research and does not aim to detail the risks involved in trading or liquidity providing associated with the automated market maker models of interest.

2 Definitions

2.1 Framework definition

We consider a set of n-assets denoted $(\alpha_i)_{i\leq n}$ and we additionally define $\beta = \alpha_n$ for writing convenience. The spot $\left(S^i_{t\geq 0}\right)_{i\leq n}$ associated with the asset pair α_i, β denotes the amount of units of β needed to buy one unit of α_i at time t. Naturally, this would imply that $S^n_t \equiv 1$ by construction at any time. In the remainder of the paper, when no additional information is provided, we assume that the numéraire of choice will be the asset β, that is all quantities will be denominated in amounts of β. Moreover, when referring to the pair α, β without specific indices, it signifies that we are working under the two-assets case where $\alpha \equiv \alpha_1$ and $\beta \equiv \alpha_2$.

In the AMM framework, and the now well established constant market maker function, it is common practice to have reserves amounts appear explicitly in the formulation and definition of the CFMM function, which as discussed in extensive details in [6] allows to link a valid trade to the reserves time evolution. While the scope of CFMM covers a large set of possible automated market maker, in this paper we will focus primarily on Uniswap V2 [4] and Balancer V1 [26], that are examples of constant product market makers. Let us denote pool reserve sizes as $\left(R^i_{t\geq 0}\right)_{i\leq n}$, a positive quantity that represents the reserve amounts of asset α_i. Where no super-script is specified, the process $\boldsymbol{R}_t$ is a n-dimensional representation of each reserve with R^i_t as element.

A trade will give rise to a constant proportion of fee denoted $(1-\gamma)$, where $0 < \gamma \leq 1$ and with $\gamma = 1$ for the case where no trading fees are considered. This means that any amount a trader is willing to sell will be scaled by γ to compute the actual input amount of the trade, which will naturally provide a lower output amount. The reserves are however updated with the total input amounts such that liquidity providers are rewarded for providing liquidity to the DEX.

Following the definition in [6], we let a trade be a tuple of vector values, $(\boldsymbol{\Lambda}, \boldsymbol{\Delta})$ where $\boldsymbol{\Lambda}$, a n-dimensional real valued vector is the output amounts resulting from the AMM following a valid trade and $\boldsymbol{\Delta}$, also a n-dimensional real valued vector is the input amounts for a given trade. Each element of the vector, that is $\left(\Lambda^i, \Delta^i\right)$ are the output and input amounts respectively of asset α_i for a specific trade. Let us write the following definition,

Definition 1. An automated market maker is a constant function market maker if and only if there exists a continuous, once differentiable with continuous derivatives function with respect to all variables, $\phi : (\mathbb{R}^+)^n \times (\mathbb{R}^+)^n \times (\mathbb{R}^+)^n \to \mathbb{R}$, such that for any given valid trade $(\boldsymbol{\Lambda}, \boldsymbol{\Delta})$ at a positive time t,

$$\phi(\boldsymbol{R}_t, \boldsymbol{\Lambda}, \boldsymbol{\Delta}) = \phi(\boldsymbol{R}_t, \mathbf{0}, \mathbf{0})\,, \tag{2.1}$$

We also denote $\psi(\boldsymbol{R}_t) = \phi(\boldsymbol{R}_t, \mathbf{0}, \mathbf{0})$.

If the trade is executed, the above formula should be understood with reserves immediately prior to the jump associated to the trade, that is $\boldsymbol{R}_{t-}$ since the value of the reserves is updated at trade time.

While continuity and differentiability is not a necessary assumption, we will assume this to be verified in the remainder of the paper. A classic example is the Balancer [26] n-assets expression which accounts for Uniswap as a limit case with,

$$\phi(\boldsymbol{R}_t, \boldsymbol{\Lambda}, \boldsymbol{\Delta}) = \prod_{i=1}^{n} \left(R^i_t + \gamma \Delta^i - \Lambda^i\right)^{w_i}, \tag{2.2}$$

where $w_i \in]0,1[$ and $\sum_{i=1}^n w_i = 1$ and where Uniswap is the duo of assets case with weights of 0.5.

Remark. In the remainder of the article, when not specified, 'Uniswap' will refer to the Uniswap V2 DEX [4], while 'Balancer' will refer to the Balancer V1 DEX [26].

2.2 Dynamical properties

Quantities defined in the previous section such as pool reserves, are jump processes and their value will change at trade times. Therefore, we define a random time set; the set of trading times $\mathcal{T} = \{\tau_1, \tau_2, ...\}$, with $(\tau_i)_{i\geq 1} \in \mathbb{R}^+ \cup \{\infty\}$. This also allows to define the processes $(\boldsymbol{\Lambda}_t)_{t\geq 0}$ and $(\boldsymbol{\Delta}_t)_{t\geq 0}$ that are zero except on trade times $t \in \mathcal{T}$ where they are linked together by (2.1). We note that a market participant performing a trade at time t can choose to either provide the input amounts $\boldsymbol{\Delta}_t$ which are asset quantities to be sold, or output amounts $\boldsymbol{\Lambda}_t$ that are asset quantities to be received. Any combination is theoretically possible for each element, namely Λ^i_t and Δ^i_t. However, a rational trader is unlikely to have both Λ^i_t and Δ^i_t to be jointly non-zero [6].

The evolution of the pool reserves can be summarized by jump processes, namely, for any $t \in \mathcal{T}$,

$$\boldsymbol{R}_t = \boldsymbol{R}_{t-} + \boldsymbol{\Delta}_t - \boldsymbol{\Lambda}_t \,. \tag{2.3}$$

2.3 Reported and effective spot price

The spot value bears a primary importance in the definition of a market. In the foreign exchange (FX) market, the spot is defined as the ratio of notional values involved in a FX cash exchange operation; that is an order to trade an amount of N^{ccy1} for an amount of N^{ccy2} specifies the spot value as $N^{\text{ccy2}}/N^{\text{ccy1}}$. The spot value of the Uniswap V2 decentralized exchange [8] follows a similar definition. Indeed, the spot is defined as the ratio of the reserves at any given time. However, for other CFMM the spot is not necessarily intuitive and is linked to the trading function ϕ. Let us provide hereafter a few definitions that will be useful for the remainder of this paper.

The reported spot $\left(S_t^i\right)_{i \in [\![1,n]\!]}$ is defined as the price of the AMM at time $t \geq 0$. As an example, the reported spot for Balancer using

$$\psi_{\text{Bal}}(\boldsymbol{R}_t) = \prod_{i=1}^{n} \left(R_t^i\right)^{w_i} , \tag{2.4}$$

leads to the ratio of the weights and pool sizes,

$$S_t^i = \frac{w_i R_t^n}{w_n R_t^i} \,. \tag{2.5}$$

For Uniswap, as $\psi_{\text{Uni}}(\boldsymbol{R}_t) = \sqrt{R_t^1}\sqrt{R_t^2}$, the reported spot is simply the ratio of the pool sizes which is similar to a FX cash trade as expected.

Finally, the effective spot $\left(\bar{S}_t^i\right)_{i \in [\![1,n]\!]}$ is the price of a trade of arbitrary size, which generally accounts for slippage, see sub-section 2.5 and for more details [15].

2.4 Impermanent loss for geometric mean markets

As discussed, liquidity providers will earn trading fees by providing their assets to the DEX. These will then be used as reserves for trading. This means that LPs make a choice to switch from a static buy-and-hold portfolio, where the amount of assets stays constant over time, to a dynamic portfolio where their assets will fluctuate according to the AMM dynamic and market movements. It is natural to compare the evolution of LPs holdings when they enter a DEX versus a buy-and-hold portfolio.

2.4.1 Impermanent loss without trading fees

Proposition 2. *The impermanent loss for a liquidity provider entering at time t_0, in a geometric mean CFMM without trading fees, is given by, for any $t \geq t_0$,*

$$IL_t = \frac{\prod_{i=1}^{n} \left(z_t^i\right)^{w_i}}{\sum_{i=1}^{n} w_i z_t^i} - 1 \,, \tag{2.6}$$

with for any $i \in [\![1, n]\!]$,

$$z_t^i = S_t^i / S_{t_0}^i \,, \tag{2.7}$$

and $IL_t \leq 0$.

Proof. The proof and additional details are provided in [15]. □

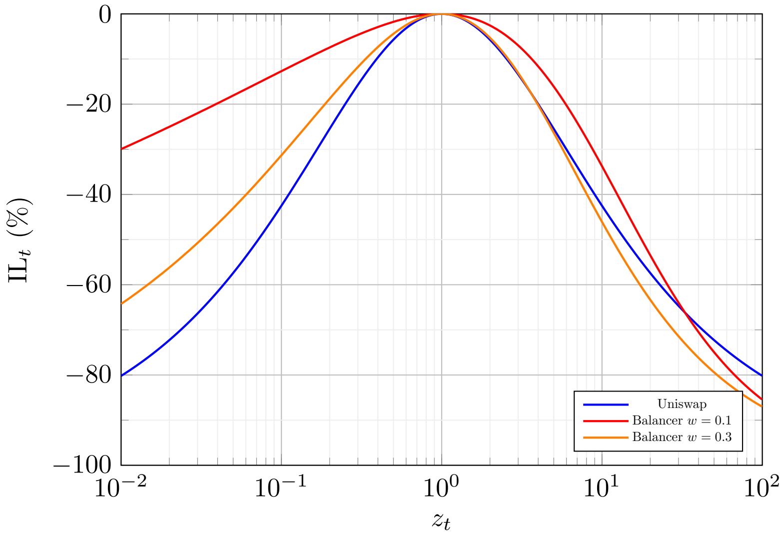

Figure 2.1: Impermanent loss under Uniswap, Balancer $w = 0.1$ and Balancer $w = 0.3$ as a function of z_t on a log-scale. The Balancer IL is improved on the downside spot movements when the weight is decreased.

For Uniswap, $n = 2$, we can define $z_t \equiv z_t^1$ and by definition, the second asset being β, we have $z_t^2 \equiv 1$. Therefore, the expression of the IL becomes,

$$\mathrm{IL}^{\mathrm{Uni}}(z_t) = \frac{2\sqrt{z_t}}{z_t + 1} - 1\,, \tag{2.8}$$

It is interesting to notice that,

$$\mathrm{IL}^{\mathrm{Uni}}(z) = \mathrm{IL}^{\mathrm{Uni}}(1/z)\,,$$

which means that under Uniswap by ignoring the trading fee, for a liquidity provider that enters the pool, the risk of loss against a buy-and-hold portfolio will be similar if the spot increases or decreases by a factor η or $1/\eta$, respectively. This symmetry suggests to plot the impermanent loss curves on a logarithmic scale.

For Balancer with two assets, we additionally introduce $w \equiv w_1$ and therefore $w_2 = 1 - w$. In this case the expression of the IL becomes,

$$\mathrm{IL}^{\text{Bal-2}}(z_t) = \frac{z_t^w}{w z_t + 1 - w} - 1\,. \tag{2.9}$$

Contrary to Uniswap where we have perfect symmetry for high and low spot returns, Balancer shows asymmetry. For $w < 0.5$ one has smaller IL for any return $z < 1$ compared to high spot regimes $z > 1$. Symmetric conclusions hold for $w > 0.5$. Therefore Balancer provides an effective solution for improving the impermanent loss on one direction of spot movement. However, if the spot deviates to the opposite side, Balancer is close to Uniswap in terms of impermanent loss. We will also see in sub-section 2.5 that Balancer with weights diverging from 0.5 does not offer good slippage properties on one trading direction.

In Figure 2.2 and 2.1 one can see the asymmetric improvement of Balancer's impermanent loss depending on the weights. For instance with $w = 0.1$, on Figure 2.2, if the spot drops by 90% compared to the entry level of the LP, Balancer offers an improvement of IL, namely from −42.5% under Uniswap to −12.7%. However, on the flip side when spot increases by a factor of 10, the improvement is smaller; from −42.5% under Uniswap to −33.7% only. For the same weight, one can also notice that the Balancer IL becomes worse than Uniswap when the spot increases by a factor greater than 30.

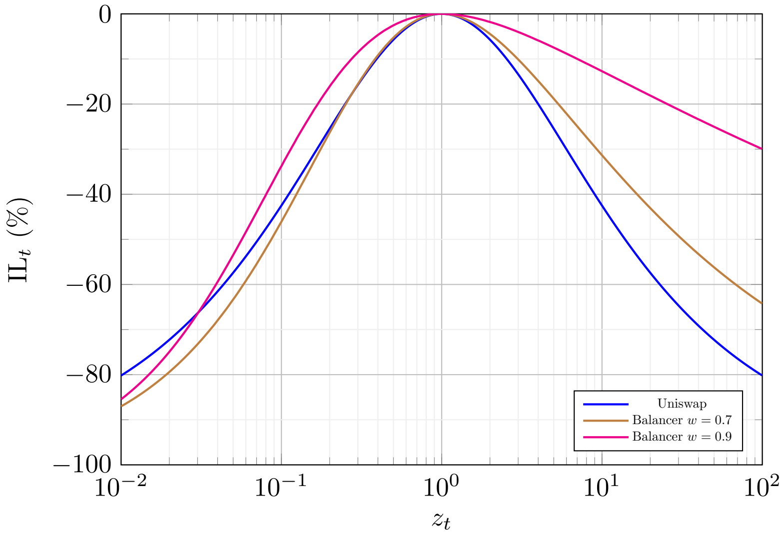

Figure 2.2: Impermanent loss under Uniswap, Balancer $w = 0.7$ and Balancer $w = 0.9$ as a function of z_t on a log-scale. The Balancer IL is improved on the upside spot movements when the weight is increased.

2.4.2 Impermanent loss with trading fees

As seen in sub-section 2.4, providing liquidity without earning trading fees will lead to a loss due to the unfavorable balance of assets as soon as the spot deviates from the entry level. Trading fees are a very efficient way of cancelling this potential loss and more importantly allowing to earn returns when sufficient trading volumes accumulate over time.

Proposition 3. *The impermanent loss for a liquidity provider entering at time t_0, in a geometric mean CFMM, with trading fees for any $t \geq t_0$, is given by,*

$$IL_t = \rho_t \frac{\prod_{i=1}^{n} \left(z_t^i\right)^{w_i}}{\sum_{i=1}^{n} w_i z_t^i} - 1 \,, \tag{2.10}$$

with for any $i \in [\![1, n]\!]$,

$$z_t^i = S_t^i / S_{t_0}^i \,, \tag{2.11}$$

and where $\rho_t \geq 1$. The parameter ρ takes into account all the trading volumes between t_0, when the liquidity provider joins the pools, and time t.

Proof. The proof and additional details are provided in [15]. □

The parameter ρ_t increases with each new trade starting from 1 at inception when the LP joins the pools. The longer the LP holds his/her position in the DEX, the higher will be the value of ρ_t. It is also worth noting that if trading volume increases, this parameter grows faster, improving the IL even more efficiently. In Figure 2.3 several profiles of IL are represented with different fixed values of ρ_t. The gradual increase of ρ_t shifts the impermanent loss profile higher and reduces the area where the IL is negative. For example, when trading fees bring ρ_t from 1 to 1.7, compared to a buy-and-hold strategy, liquidity providing remains profitable even when the spot changes by a factor of 10 (either increase or decrease). The profile of IL obtained with $\rho_t \equiv 1$ corresponds to the case with no trading commission described in the previous section.

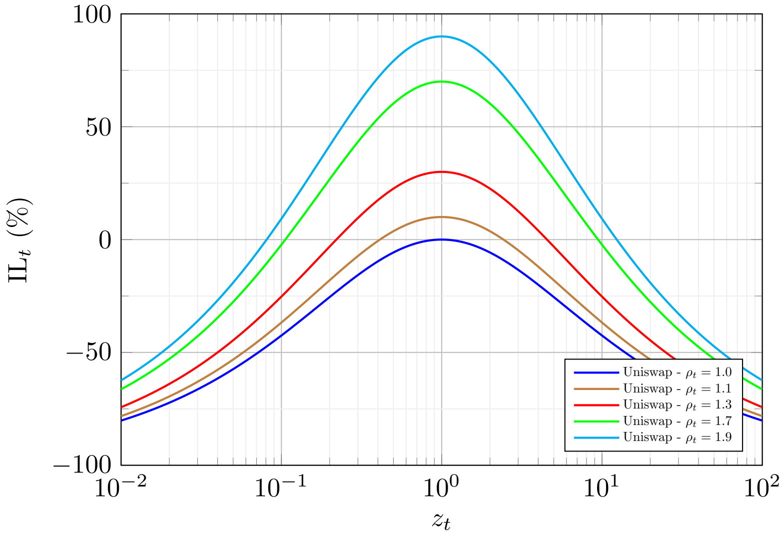

Figure 2.3: Impact of trading fees on the profile of impermanent loss for a LP joining a DEX similar to Uniswap. The accrual of commissions increases ρ_t gradually. Different IL profiles are represented for $\rho_t = \{1, 1.1, 1.3, 1.7, 1.9\}$. $\rho_t = 1$ corresponds to the case without trading fees. The x-axis is z_t on a log-scale.

2.5 Slippage

Slippage is the difference between the price at which a trade is expected to get executed and the actual price at which it occurs. With exchanges relying on order books [18], slippage can be positive or negative, depending on whether the difference is favorable or not. For DEXs, such as Uniswap or Balancer, the prices depend on the liquidity pools sizes which have slower dynamic properties than order books. The resulting slippage is often unfavorable for the trader.

We are defining the slippage as the difference of the effective spot of asset i, $\bar{S}_t^i$, which is the price of a trade of arbitrary size and its associated marginal price . Other definitions or views are possible as well. Additionally, and to simplify reasoning, we ignore the impact of trading fees such that, the marginal price is equal to the reported price and we can write the slippage Υ_t without trading fees as,

$$\Upsilon_t = \frac{\bar{S}_t^i - S_t^i}{S_t^i} \,.$$

Details about slippage calculation are provided in [15].

On Figure 2.4, both functions $\Upsilon^{\beta\to\alpha} : \mathbb{R}^+ \to \mathbb{R}^-$ and $\Upsilon^{\alpha\to\beta} : \mathbb{R}^+ \to \mathbb{R}^-$ are represented on the same graphic, where negative abscissas are mapped to $x \to$ -$\Upsilon^{\beta\to\alpha}(-x)$ and positive abscissas, mapped to $x \to \Upsilon^{\alpha\to\beta}(x)$.

One can see that Balancer with weights different from 0.5, offers very asymmetric behavior in terms of slippage depending on the trade direction. For example low weights offers low slippage when selling α for β. However from β to α, the slippage is much higher than Uniswap. The situation is symmetric for high weights. We remind here that Balancer with asymmetric weights helps improve the IL for one direction of spot movement but not both. In conclusion, using Balancer with low or high weights partially improves the IL either on the increase or decrease of the spot but also offers very asymmetric slippage depending on the trade direction.

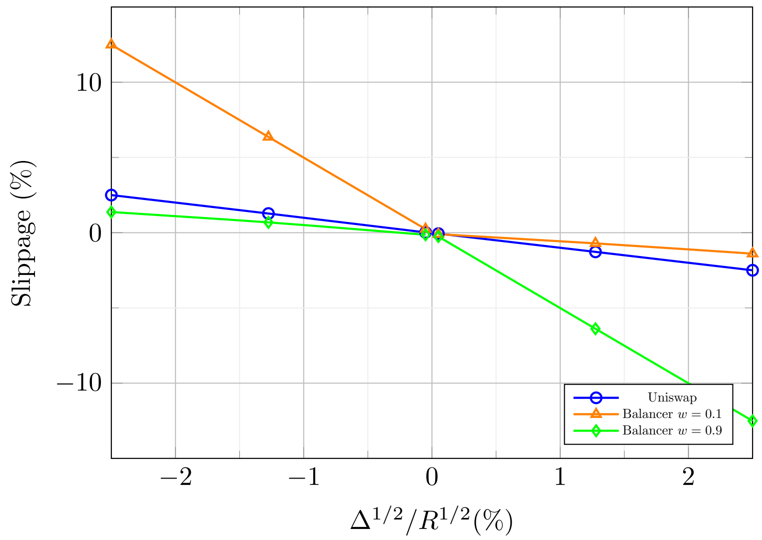

Figure 2.4: Slippage for geometric mean markets, with Uniswap and Balancer $w = \{0.1, 0.9\}$ as a function of Δ^1/R^1 for positive values on the abscissa and $-\Delta^2/R^2$ for negative values on the abscissa. The ordinate is the slippage for a trade to sell α for β on positive abscissa values. Similarly, the ordinate is - slippage for a trade to sell β for α on negative abscissa values. One can see symmetrical slippage for Uniswap for both trade directions but asymmetric slippage for each Balancer. For example, Balancer with weight 0.1 is favorable for traders who sell α for β.

3 AllianceDEX Synchronized AMM

In this article, we introduce a new type of AMM which combines the properties of several CFMMs together. The goal is two-fold, on the one hand this approach allows to gain flexibility on the features of the resulting AMM, on the other hand, as a direct application of this technique and driving result of the paper, we propose an AMM with strongly reduced downside risk for liquidity providers. The latter comes from the interesting properties of geometric mean markets CFMM with asymmetric weights which improve the impermanent loss either on the rise or on the fall of the spot, depending on the value of the weights of the geometric mean as seen in sub-section 2.4.1. Additional details about synchronized AMM are provided in [15].

To avoid complex notations, we focus on the two-CFFM synchronized AMM. This bears similarities with the two-pools Balancer with aligned spot prices discussed in [7]. In sub-section 2.1, we have denoted $(\boldsymbol{R}_t)_{t\geq 0}$ the reserves for a given CFMM, which we will use here to account for the reserves of a first CFMM $(\mathcal{G}_1)$. Therefore, we also denote $\left(\bar{\boldsymbol{R}}_t\right)_{t\geq 0}$ the reserve process for a second CFMM $(\mathcal{G}_2)$. Both $\mathcal{G}_1$ and $\mathcal{G}_2$ are defined by their associated constant function $\phi_1(\boldsymbol{R}_t, \cdot, \cdot)$ and $\phi_2\left(\bar{\boldsymbol{R}}_t, \cdot, \cdot\right)$ respectively.

Additionally, the reported spots before and after the trade should match perfectly. Without loss of generality, since the reported spots of both $\mathcal{G}_1$ and $\mathcal{G}_2$ are matching, we will denote the reported spot of the synchronized AMM by $(\boldsymbol{S}_t)_{t\geq 0}$, following the reported spot notation of $\mathcal{G}_1$.

3.1 Two-assets synchronized geometric mean AMM

In this section, we consider a two assets AMM built with the synchronization of two geometric mean CFMM. In the remainder of the article we refer to this automated market maker as sync-AMM. There are two reasons behind this choice; the first is that two assets within two synchronized CFMM is a natural configuration for such a system and therefore also the easiest both numerically and theoretically. The second and most important reason is that by combining two geometric CFMM, with well chosen weights, we can take advantage of their best properties and combine them to improve the impermanent loss for LPs. It is important here to pin-point the fact that traders will not be able to interact with the two internals CFMM, as this will be done following the procedure below, and will result in a standard AMM from an end-user perspective. The trade splitting and rerouting into the two internal CFMM as well as the synchronization of the reported spots is handled by the new type of DEX.

This section derives the main result of this article. The two-assets synchronized geometric mean CFMM are discussed in extensive details in [15] and summarized in the remainder of this paper.

We write,

$$\psi_1(\boldsymbol{R}_t) = \left(R_t^1\right)^w \left(R_t^2\right)^{1-w}, \qquad \psi_2(\bar{\boldsymbol{R}}_t) = \left(\bar{R}_t^1\right)^{\bar{w}} \left(\bar{R}_t^2\right)^{1-\bar{w}}, \tag{3.1}$$

where the driving weights of the geometric mean $\mathcal{G}_1$ and $\mathcal{G}_2$ are w and $\bar{w}$ respectively. In order to ensure that the underlying reported spots of $\mathcal{G}_1$ and $\mathcal{G}_2$ are matching we require, for any t,

$$\frac{w}{1-w}\frac{R_t^2}{R_t^1} = \frac{\bar{w}}{1-\bar{w}}\frac{\bar{R}_t^2}{\bar{R}_t^1}. \tag{3.2}$$

Finally, while most of the derivations and discussions below can be made under the assumptions that w and $\bar{w}$ are not linked together. In our case, we will work with

$$\bar{w} = 1 - w\,. \tag{3.3}$$

3.1.1 Impermanent loss of a synchronized AMM without trading fees

We consider here the case where the AMM accounts for two assets $\alpha \equiv \alpha_1$ and $\beta \equiv \alpha_2$ and discuss the impermanent loss for the sync-AMM discussed in Section 3.1 in the case where there is no trading fees.

As seen on Figure 3.1, the new AMM takes advantages of the best features of each CFMM and helps improve the IL on both increase and decrease of the spot performance. For example, the synchronized AMM with $w = 0.1$, leads to an IL of -16.3% against -42.5% for Uniswap if the spot either increases or decreases by a factor of 10 with respect to the LP entry value. The IL for the new AMM is perfectly symmetric as chosen in (3.3).

The gradual accumulation of trading fees combined with this new profile of impermanent loss is the building-block of improved returns for liquidity providers. More details and associated simulations are provided in Section 4.

4 Simulations and numerical results

In this section, we discuss the simulation of the synchronized two-assets AMM introduced in sub-section 3.1. First and foremost we will use the following simplified market model; we suppose the market spot$(Z_t)_{t\geq 0}$ representing one unit of α in terms of β, follows a geometric Brownian motion stochastic differential equation under the real-world probability measure $\mathbb{P}$ with,

$$\begin{cases} dZ_t &= \mu Z_t dt + \sigma Z_t dW_t \\ Z_0 &\geq 0, \end{cases} \tag{4.1}$$

where W is a Brownian motion under $\mathbb{P}$, and where $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$. The diffusion parameters μ and σ are not calibrated to the market but arbitrarily chosen values which align with the high growth-rate and volatility sometimes encountered on crypto-currency markets. For example, over the second half of March 2021, Bitcoin realized volatility ranged from 70% to 85%, and similarly Ethereum realized volatility ranged from 60% to 95% (source [2]). More precisely, we present the set of results obtained with the below driving numbers,

$$Z_0 = 137, \quad \mu = 25\%, \quad \sigma = 120\%.$$

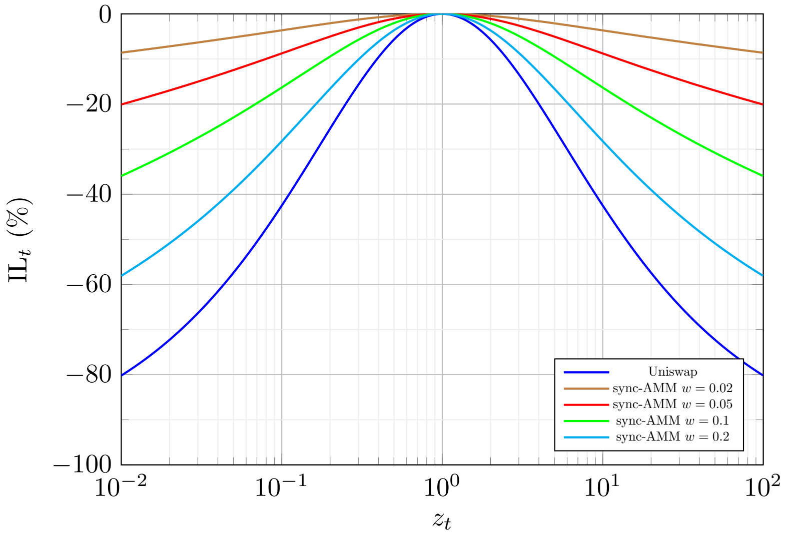

Figure 3.1: Impermanent loss without trading fees for LP under Uniswap and Sync-AMM with different values of w, namely $w = \{0.02, 0.05, 0.1, 0.2\}$. The plot is as a function of z_t on a log-scale.

The trading fees are assumed fixed at 0.3% per trade and the simulations are run over periods of 3 years with an average amount of 50 000 trades per year, excluding arbitrage trades.

The state variables of the AMMs simulated are calculated following details from sub-section 2.2. Extensive details about Monte Carlo simulations as well as sampling procedure for both (4.1) and random times as described in sub-section 4.1 can be found in [17].

4.1 Trade generation and arbitrage

Our assumption for the different simulations is that trades are simulated such that inter-arrival times follow an exponential distribution of mean $\frac{1}{\lambda}$, which provides an average of λ jumps per unit of time. We chose an average of 50 000 trades per year, randomly happening on the timeline. Let $t \in \mathcal{T}$ be a trade time, the trade size percentage η_p associated to the trade is defined as a Gaussian random variable with mean 0 and standard deviation $p/4$, where resulting sizes percentages are truncated to stay within the bounds,

$$[-p, p],$$

where $p = 1\%$ in our simulations. We denote for any $t \geq 0$,

$$N_t^1 = R_t^1 + \bar{R}_t^1, \quad N_t^2 = R_t^2 + \bar{R}_t^2.$$

A positive trade size percentage η_p is assumed to be a trade to buy or sell asset α for asset β, whereas a negative value, is a trade to buy or sell asset β for asset α. The buy or sell order choice is determined randomly with equal probability. A trade on α therefore will be of size $|\eta_p|N_t^1$ and a trade on β will be of size $|\eta_p|N_t^2$.

The trade generation discussed above accounts for any spontaneous trades done by traders but does not account for a quoted market spot, e.g. the spot value available on a centralized exchange. Where a market spot is available outside of the sync-AMM, it is possible for arbitrage opportunities to arise, and we assume that in between any two trades, an arbitrageur will try to take advantage of the AMM reported spot divergence with the market spot.

4.2 Liquidity addition and withdrawal

Our assumptions are simplistic but provide a good model for the analysis of the liquidity provider returns, e.g. impermanent loss.

We will work with an initial liquidity amount, brought by a first liquidity provider at time $t = 0$,

$$N_0^1 = 10\,000\,000, \quad N_0^2 = Z_0 N_0^1 = 1\,370\,000\,000.$$

Each subsequent liquidity provider will bring precisely 5% of the initial reserve size N_0^1 when he/she joins the sync-AMM. The liquidity provider will also deposit $Z_0 N_0^1$ into the reserve of β as part of the liquidity addition. Withdrawal can be done in full or partially by the LP in both of the underlying synchronized CFMM. For ease of testing and showcase, we assume that the user exits fully from both synchronized CFMM, at the same time. The associated return is discussed hereafter and displays both the impact of impermanent loss and the rewards thanks to trading fees.

Over a $3Y$ time frame, 20 new liquidity providers will enter the AMM with entry times uniformly distributed over the timeline. While this assumption is simplistic, it allows to validate the dynamics of the DEX.

4.3 Liquidity provider returns

When entering the AMM, the liquidity provider returns, which we will refer to as impermanent loss, can be calculated at every point in time. More specifically, since the liquidity provider can decide to exit the DEX at any given time, it is possible to evaluate the profit-and-loss of the holdings compared to a simple buy-and-hold strategy started at the LP entry time in the DEX. This without the need for the LP to effectively exit the liquidity pools, which allows to assess the robustness of impermanent loss over different exit points, therefore provides a large scope of possible outcomes.

4.4 Numerical results

The aggregated results for the liquidity provider returns and impermanent loss are computed with 30 paths. While this is low to achieve any reasonable convergence properties on the market spot distribution, this amount of paths combined with the number of possible exit points allows the gathering of a considerable amount of information and scenario outcomes on the liquidity provider PnL and IL. Exit points can happen and profit-and-loss can be calculated at every time step, which will range between 50 000 and 100 000 times per year with the lower bound being the amount of trades independent of arbitrage and the upper bound which accounts for the extra amount of trades a year attributed to arbitrage.

Out of the total number of paths, we hereafter extract 4 of them to display dynamic behaviors. In Figures 4.1 4.2 4.3 4.4 we display the time series of the market spot as well as the sync-AMM reported spot. We can see that the spot moves in a large range of values which provides useful stress test cases.

We also display in Figure 4.5 4.6 4.7 4.8 the impermanent loss of all LPs who entered the given AMM at different time and therefore different spot values. Consequently, they are exposed to different effects of market moves and their profit-and-loss will naturally look different. We note that an important feature of the sync-AMM, which is shared by most CFMM such as Uniswap or Balancer, is that liquidity addition or withdrawal does not impact the PnL of other liquidity providers within the DEX.

Remark. The various colors in Figures 4.5 4.6 4.7 4.8 correspond to the IL of different liquidity providers, entering at different times as well as on possibly different paths, which provides an aggregated view of a large amount of possible outcomes and scenarios.

4.4.1 Time series under stressed markets

In addition to the above, in Table 2, we display the values of the IL with trading fees, i.e. the liquidity providers returns with trading fees compared with a buy-and-hold strategy started time $t = 0$. One can notice that

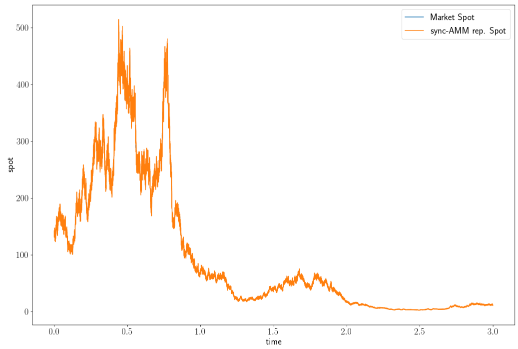

Figure 4.1: Paths 1: Market spot Z_t, sync-AMM reported spot S_t. X axis is the time in years.

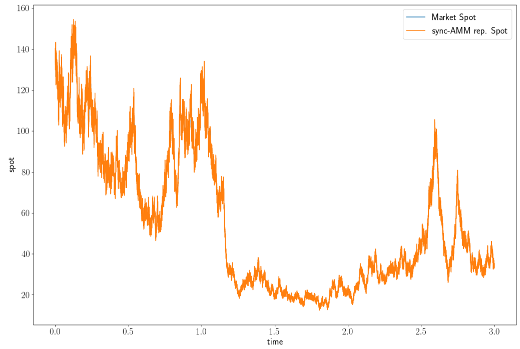

Figure 4.2: Paths 2: Market spot Z_t, sync-AMM reported spot S_t. X axis is the time in years.

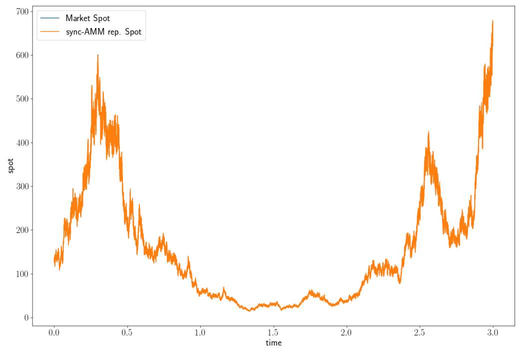

Figure 4.3: Paths 3: Market spot Z_t, sync-AMM reported spot S_t. X axis is the time in years.

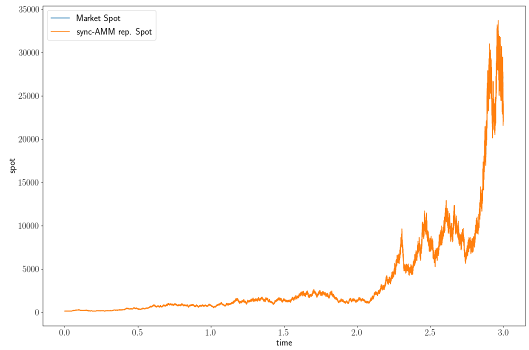

Figure 4.4: Paths 4: Market spot Z_t, sync-AMM reported spot S_t. X axis is the time in years.

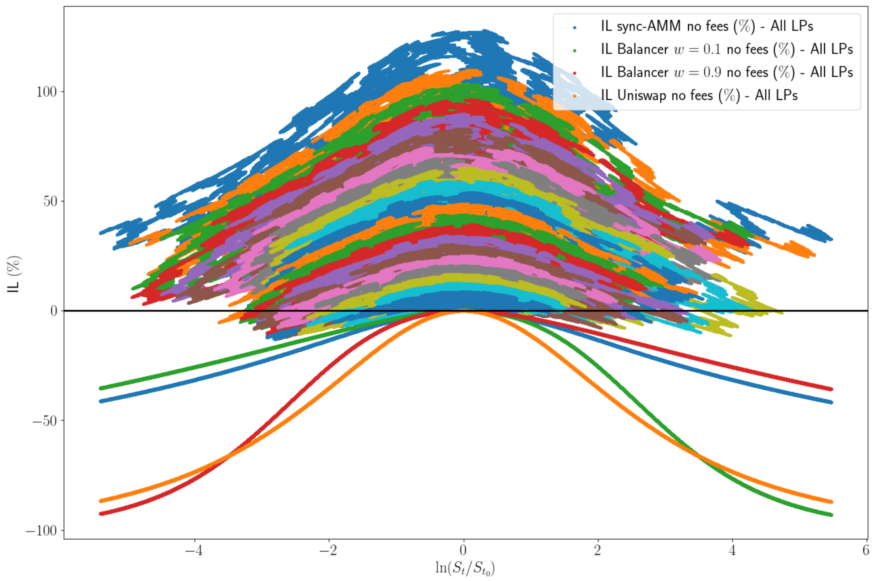

Figure 4.5: Sync-AMM $w = 0.1$: Impermanent loss with trading fees for all liquidity providers who entered the AMM. Each color corresponds to a given liquidity provider and path.

in both cases (when the initial spot increases or decreases by a factor of 100), the IL of the sync-AMM is considerably better than on Uniswap and either a Balancer with low weight or high weight depending on the spot deviation direction. Because of the latest remark, the sync-AMM is then consistent both on the rise and fall of the spot which provides a strong improvement of impermanent loss in the two scenarios. As a benchmark, we also provide the impermanent loss without trading fees in Table 1 in which we notice a similar behavior where the sync-AMM provides consistent improvement compared to Uniswap in both up- and down-trend scenarios.

In Figure 4.9, we display the same sample paths as per Table 2 but with the entire time-series of the liquidity providers returns where it is possible to display more precisely the different changes over time of the impermanent loss with the addition of trading fees. The sync-AMM improves the liquidity provider returns, even when large swings of spot prices occur.

4.4.2 ALBT–USDT and ALBT–ETH back-test

In order to provide a real case study, we hereby compute and display the liquidity provider returns with trading fees for the token pairs ALBT–USDT and ALBT–ETH where liquidity pools are hosted on Uniswap. We provide the returns for both Uniswap V2 and the AllianceDEX sync-AMM for a liquidity provider that would enter both DEXs on the 11th of November 2020. The liquidity provider returns account for both the impermanent loss and the rewards from trading fees and is referred to as IL with trading fees in Figures 4.10 and 4.11.

Figure 4.10 shows that over a short-period of a few months, the spot moves from 0.052 USD to a maximum of 0.965 USD and finally reaches 0.37 USD towards the end of the testing period. This very large variation (1850% spot increase when the spot reached the maximum of 0.965 USD) over a short time-frame represents a

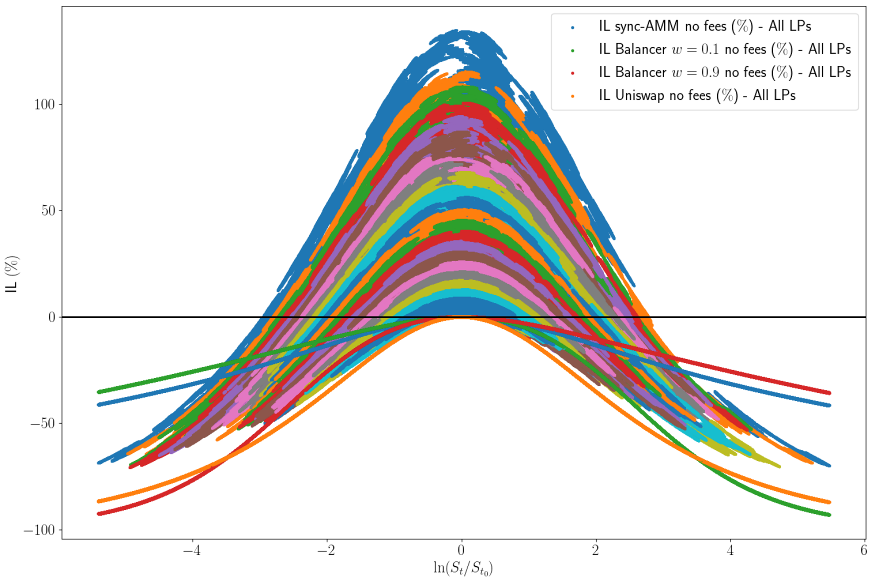

Figure 4.6: Uniswap: Impermanent loss with trading fees for all liquidity providers who entered the AMM. Each color corresponds to a given liquidity provider and path

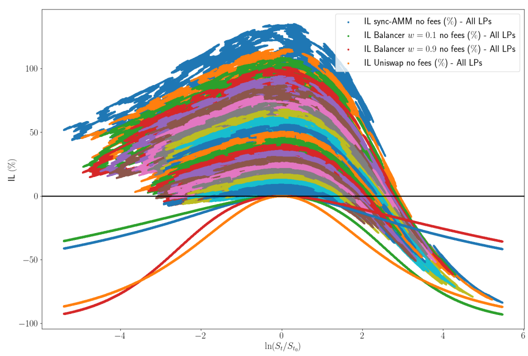

Figure 4.7: Balancer $w = 0.1$: Impermanent loss with trading fees for all liquidity providers who entered the AMM. Each color corresponds to a given liquidity provider and path

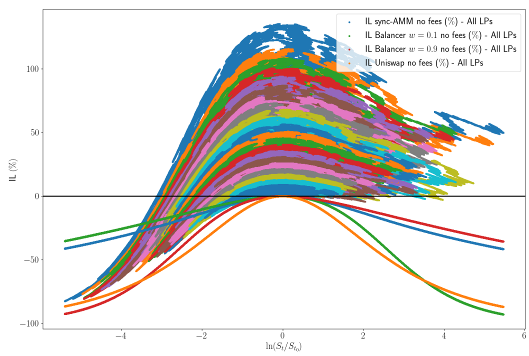

Figure 4.8: Balancer $w = 0.9$: Impermanent loss with trading fees for all liquidity providers who entered the AMM. Each color corresponds to a given liquidity provider and path

	LP returns: IL with trading fees								
	Downward trend					Upward trend			
Time	**3Y**	**2Y**	**1Y**	**6M**	**Start**	**6M**	**1Y**	**2Y**	**3Y**
Spot (ratio)	1.35 (/101.5)	3.79 (/36.1)	31.14 (/4.4)	31.05 (/4.4)	137 ($\times$1)	1 119.59 ($\times$8.2)	1 693.07 ($\times$12.4)	5 210.34 ($\times$38.0)	14 260.38 ($\times$104.1)
IL AllianceDEX	48.24%	25.04%	20.81%	5.21%	0%	−1.84%	7.41%	23.61%	46.24%
IL Uniswap	−53.56%	−42.78%	3.10%	−10.46%	0%	−28.26%	−29.72%	−44.34%	−54.24%
IL Balancer $w = 0.1$	65.18%	36.91%	24.42%	7.79%	0%	−17.21%	−19.39%	−46.11%	−66.71%
IL Balancer $w = 0.9$	−66.05%	−44.01%	14.60%	−0.34%	0%	2.25%	13.61%	35.61%	63.84%
IL Balancer $w = 0.33$	−24.01%	−20.55%	9.20%	−5.17%	0%	−31.67%	−35.48%	−55.88%	−68.92%
IL Balancer $w = 0.67$	−68.38%	−54.31%	1.71%	−11.53%	0%	−19.07%	−16.75%	−22.13%	−24.89%

Table 1: Liquidity provider returns *with trading fees* compared to a buy-and-hold strategy, i.e. "impermanent loss with trading fees". The initial spot value is $S_0 = 137$; the right-hand side of the table displays a path that increases significantly whereas the left-hand side displays a path that decreases significantly.

	LP returns: IL without trading fees								
	Downward trend					Upward trend			
Time	**3Y**	**2Y**	**1Y**	**6M**	**Start**	**6M**	**1Y**	**2Y**	**3Y**
Spot (ratio)	1.35 (/101.5)	3.79 (/36.1)	31.14 (/4.4)	31.05 (/4.4)	137 ($\times$1)	1 119.59 ($\times$8.2)	1 693.07 ($\times$12.4)	5 210.34 ($\times$38.0)	14 260.38 ($\times$104.1)
IL AllianceDEX	−36.02%	−28.15%	−8.28%	−8.26%	0%	−14.36%	−18.32%	−28.65%	−36.22%
IL Uniswap	−80.27%	−67.59%	−22.38%	−22.30%	0%	−37.74%	−47.12%	−68.50%	−80.55%
IL Balancer $w = 0.1$	−30.02%	−22.60%	−6.57%	−6.54%	0%	−28.23%	−39.48%	−69.55%	−85.90%
IL Balancer $w = 0.9$	−85.55%	−68.24%	−13.51%	−13.45%	0%	−11.17%	−14.27%	−23.05%	−30.23%
IL Balancer $w = 0.33$	−67.59%	−54.90%	−17.73%	−17.67%	0%	−40.68%	−51.44%	−74.98%	−86.75%
IL Balancer $w = 0.67$	−86.49%	−74.03%	−23.23%	−23.15%	0%	−29.67%	−37.20%	−55.75%	−67.90%

Table 2: Liquidity provider returns *without trading fees* compared to a buy-and-hold strategy, i.e. "impermanent loss without trading fees". The initial spot value is $S_0 = 137$; the right-hand side of the table displays a path that increases significantly whereas the left-hand side displays a path that decreases significantly.

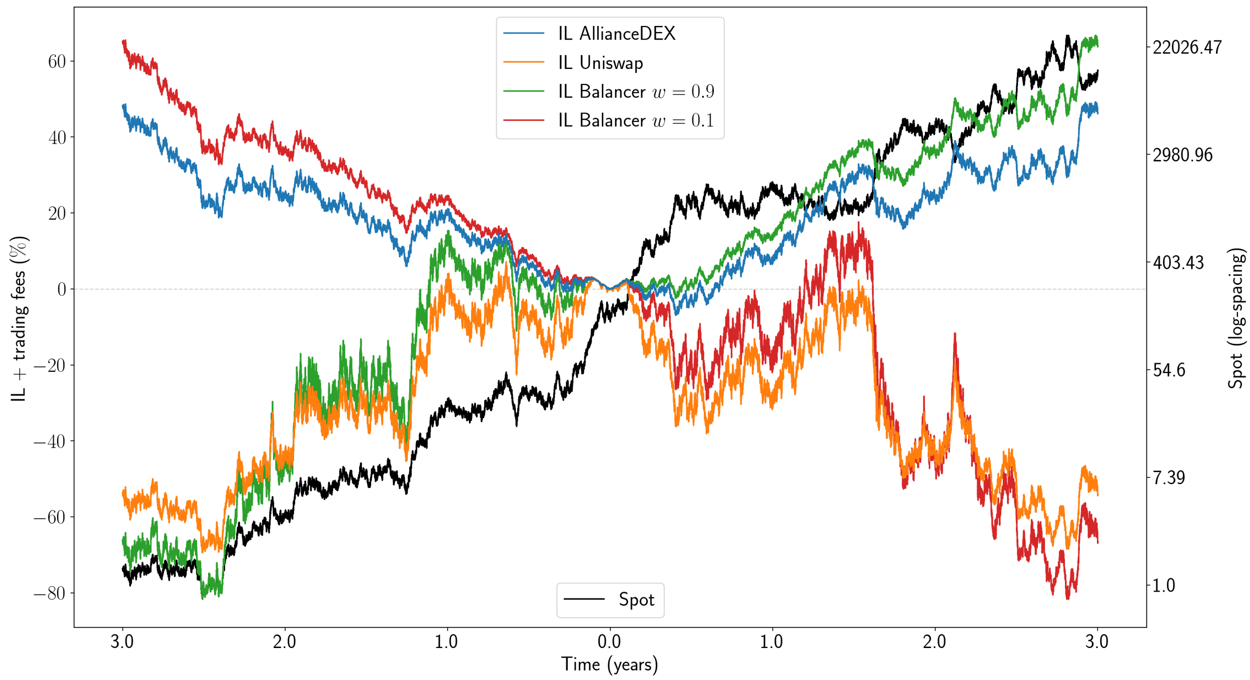

Figure 4.9: Liquidity provider returns *with trading fees* compared to a buy-and-hold strategy, i.e. “impermanent loss with trading fees”. The initial spot value is $S_0 = 137$; the right-hand side of the graphic displays a path that increases significantly whereas the left-hand side displays a path that decreases significantly.

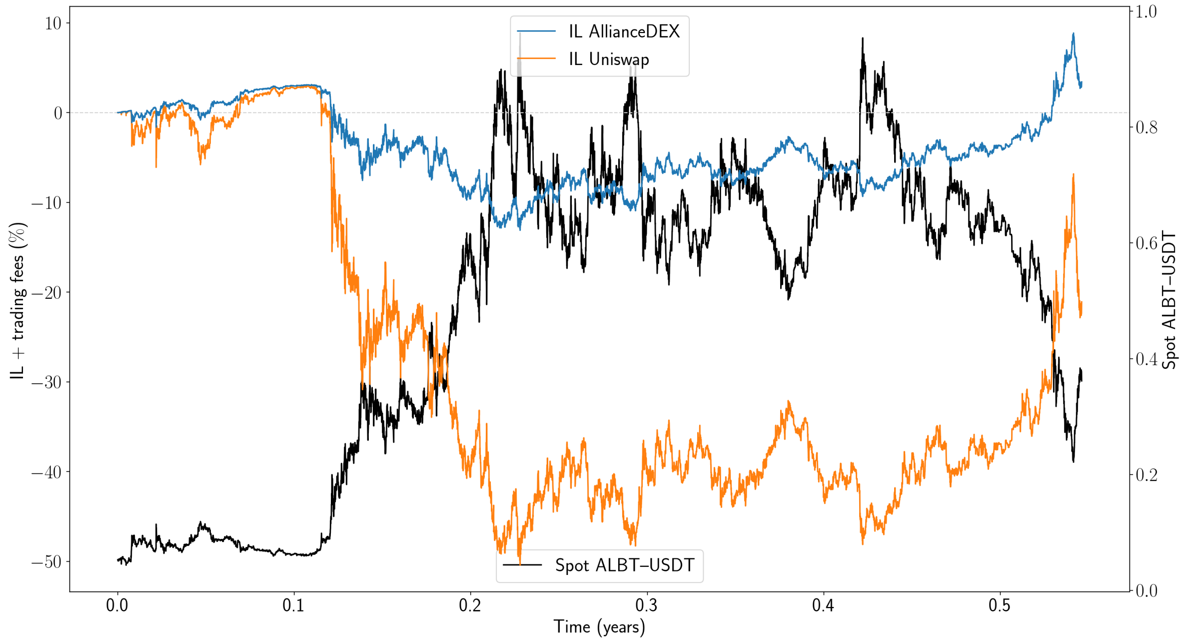

Figure 4.10: ABLT–USDT Back-test of impermanent loss *with trading fees* over a time-interval starting on the 11th of November 2020 and ending on the 25th of May 2021.

good stress test to compare the behaviors of the different DEXs at play. While on Uniswap V2 the returns compared to a buy-and-hold strategy leads to a maximum draw-down of -51%, the sync-AMM was able to considerably improve this figure with a maximum draw-down of only -14%.

Figure 4.11, although showing less drastic spot deviations from the entry-point, displays two interesting conclusions. On the one hand, after a couple of weeks, the returns of the sync-AMM are always positive along the time-line, contrary to Uniswap V2 which on a few instances, led to negative returns of around -15% due to spot moves. While towards the end of the time-frame the spot reverts back close to its original value, therefore strongly reducing the IL, the maximum difference in returns between Uniswap and the sync-AMM reaches more than 20% around the three months mark as well.

4.4.3 Slippage

In Figures 4.12 , 4.13 , 4.14 and 4.15 we display the slippage of the sync-AMM DEX which can be compared to the ones of Uniswap as well as Balancer. The impact of trading fees can be observed as the gap at the origin which shifts all curves above or below the x-axis by the fee amount. The average slippage becomes symmetrical in the sync-AMM compared to Balancer which displays a highly asymmetric behavior when weights are either small or close to 1. This feature favors trading in both directions as opposed to Balancer in such cases. Additionally, an interesting feature of the sync-AMM is a slippage that depends on the current market and reserve states and not only on the ratio of the trade size with respect to the associated reserve size like geometric mean markets CFMM. This feature displays the slippage as an apparently random value between bounded ranges when plotted with respect to the trade and reserve size ratio. That contrasts with Balancer or Uniswap, where the slippage has a functional dependence on the same ratio. Additionally, if the individual reserves of pools $\mathcal{G}_1$ and $\mathcal{G}_2$ were not visible publicly, slippage would appear stochastic and potentially could help making front-running more challenging. For the same volume of liquidity, Balancer and sync-AMM have higher slippage than Uniswap, however, if the reserve sizes were increased by the appropriate factor compared

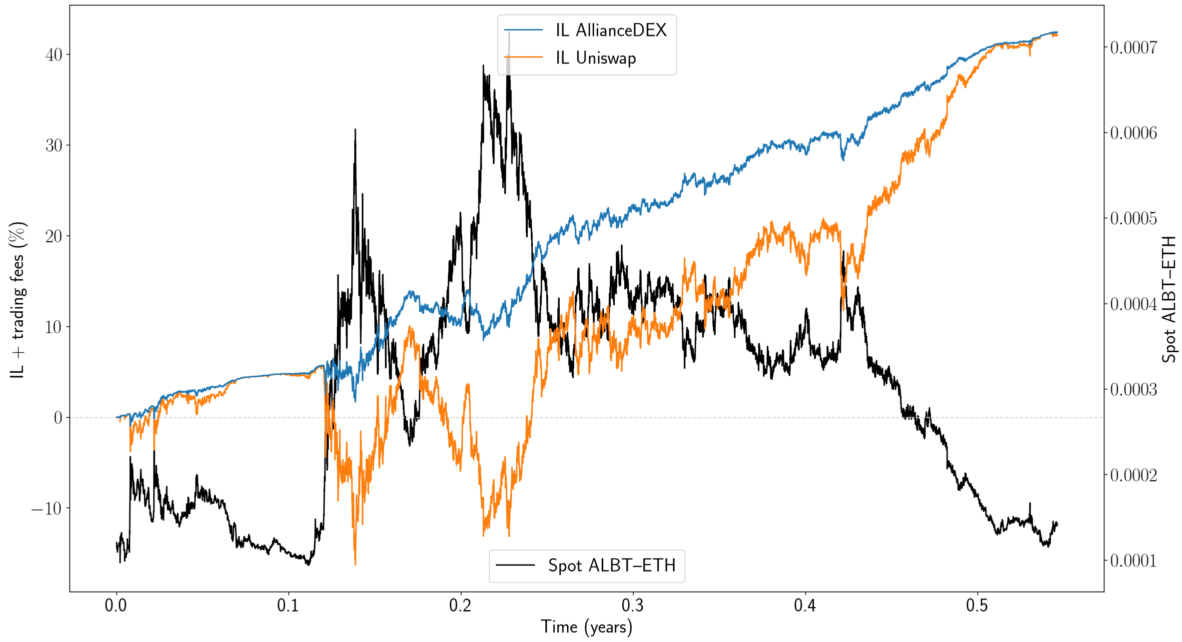

Figure 4.11: ABLT–ETH Back-test of impermanent loss *with trading fees* over a time-interval starting on the 11th of November 2020 ending on the 25th of May 2021.

to the reserves in a Uniswap-type AMM, then the average slippage would be comparable between the DEXs, while preserving the improved impermanent loss as discussed in previous sections.

5 Conclusion

With the recent innovations in the DEX industry and decentralized finance more generally, new aspects of risk exposures have emerged for investors providing liquidity to decentralized exchanges. In particular, the impermanent loss, or the risk of loss compared to a buy-and-hold strategy when the spot deviates largely from the LP's entry-point, has been a topic of strong research and development in the field. Up until now, tackling the impermanent loss problem has relied mainly on extra-features which do not modify the intricacies of the AMM itself. In this article, we focused on the underlying mechanisms of the market maker by proposing a new approach combining properties of different CFMMs. The AllianceDEX combines two dual-asset geometric mean markets with symmetric weights and provides an improvement of the impermanent loss in every spot market scenario resulting in an average increase of the returns for liquidity providers. This setup brought us to discuss impermanent loss in geometric mean markets used by well known DEXs, such as Uniswap V2 and Balancer V1 where we provided analytical formulas of the IL for the generic case of $n-$assets with and without trading fees. Our numerical results show that the sync-AMM combined with the usual 0.3% trading fees, lead to positive returns in a large amount of market scenarios, even when the spot deviates drastically from the liquidity providers entry points. Striking examples, are sample paths which lead to an increase or decrease of the asset price by a factor of 150 and where, in that instance and on the paths analyzed, the sync-AMM still provided positive returns, compared to a buy-and-hold portfolio. The symmetrical average slippage makes the proposed DEX suitable for trading in both directions as opposed to Balancer with uneven weights. This feature combined with the IL improvement should, in principle, attract liquidity providers and hence increase the available reserves which as a consequence, will reduce the slippage making the proposed solution beneficial

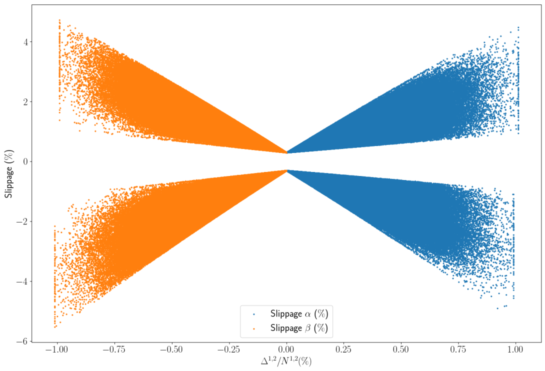

Figure 4.12: Slippage of the sync-AMM with weight $w = 0.1$.

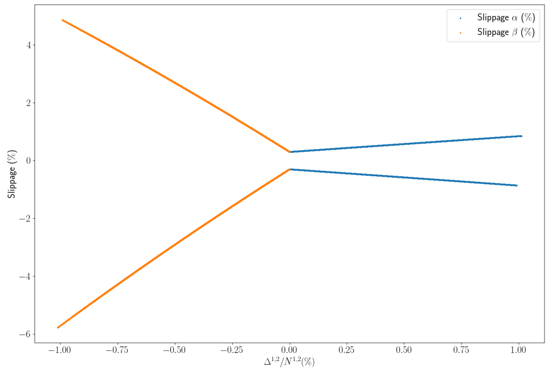

Figure 4.13: Slippage of Balancer with weight $w = 0.1$.

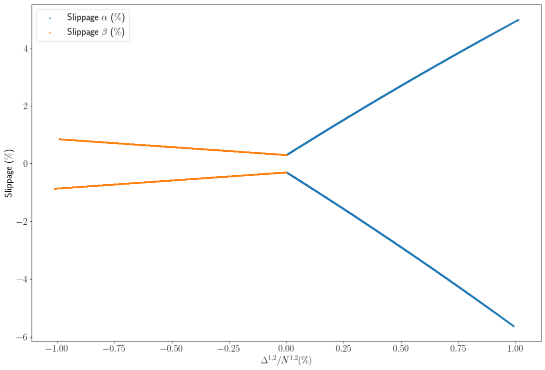

Figure 4.14: Slippage of Balancer with weight $w = 0.9$.

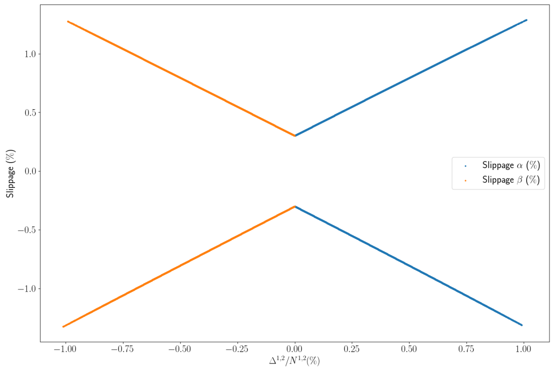

Figure 4.15: Slippage of Uniswap.

for both LPs and traders long-term. Finally this new approach to automated market making could potentially open other research topics on additional improvements of the impermanent loss, reduction of slippage costs as well as new features such as single-sided liquidity provisioning.

5.1 Next steps

Extension of this work and the decentralized exchange will include a new approach to single sided liquidity provisioning. This will allow investors to deposit funds in the token of their choice instead of the pair of assets. The tokenomics will be released in an upcoming document posterior to the delivery on the testnet of the DEX. AllianceBlock will also provide new features in the next steps, namely, a treasury contract associated with yield and insurance strategies, rewards associated with liquidity provisioning risks as well as a liquidity layer optimizer. More details about these features will be released by AllianceBlock in due course.

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